1. INTRODUCTION

In the Netherlands, the Traffic and Transport Research Centre (in Dutch: Adviesdienst Verkeer en Vervoer) of the Ministry of Transport, Public Works and Water Management keep track of average traffic volumes on motorways. This is done in view of the planning of road maintenance, the observation of traffic performance, the identification of trends and future bottlenecks, etc. Data are stored in the form of flow-profiles. These profiles represent average hourly volumes, averaged per month, day and hour of the day.

Actually, traffic can only be counted on 870 of the 6200 links of the Dutch motorway network. These links are referred to as the observed links. For the other links, an estimate of the flow profile should be made. At the moment these estimates are made by multiplying a prior estimate by a growth factor. However the quality of the prior estimates should be questioned. Also it is envisaged that in the future the set of observed links will be modified. It is desirable to be able to judge the effect of such a modification on the quality of the flow profile estimates in advance.

In order to address these problems we have been commissioned to develop a method that can be used to estimate flow profiles for unobserved links. Using this method it can be determined to what extent the removal or addition of an observed link influences the error of estimation that applies to the estimates of flow profiles.

Earlier projects were aimed to extrapolate traffic data in time (BGC, 1988), or to estimate missing data in time series of data (Swaving and De Vries, 1996; De Vries en Praagman, 1995), or to identify road sections with comparable flow patterns (TRANSPUTE, 1994). Contrary to these statistically oriented approaches, this project uses the spatial dependencies of traffic flows as a basis for the estimation of flow profiles. This implies that the link volumes are considered as sums of route flows. This provides a basis to identify relations between observed and unobserved flows or better, to express the unobserved flows in the observed ones.

Although this problem is relatively unknown in literature, it is strongly connected to the well-known problem of estimating Origin-Destination (OD) matrices from traffic counts. The estimation of an OD matrix from traffic counts under the assumption of certain route choice proportions automatically implies an estimate of all link flows in the network, including the unobserved ones. Another approach is to maximize entropy...
under the constraints of the given flow observations. In (Sherali et. Al, 1994) an approach is described minimizing the total traveled distance on a network under the constraint of the observed flows. The minimization was performed with Linear Programming techniques.

When modelling the spatial interconnection of network flows, a prior OD matrix provides important information. For the Dutch context such a matrix can be imported from the National Mobility Model (in Dutch: the ‘Landelijk Model Systeem’, Rijkswaterstaat & Hague Consultancy Group, 1990). A main characteristic of the problem is however that the flow profiles consist of estimates per hour, type of day and month, whereas the available OD matrix only distinguishes between peak and off-peak, and between weekday and non-weekday.

In view of the above it was concluded that none of the methods known from literature can be applied directly to the current problem. This paper investigates three possible approaches. The approach elaborated in more detail, is the derivation of local OD matrices for small subnetworks that contain the unobserved link, followed by the estimation of the flow profile for this link based on the flows observed on its surrounding links.

2. PROBLEM DEFINITION

The objective is to estimate flow profiles for the 6200 links of the Dutch motorway network. Because these profiles can not be directly observed, they are estimated using observed flows and other data. The following input can be used:

- **Observed flows.** Observed flows are either aggregated per day or per hour, depending on the road side hardware. Also observations can be subdivided in *permanent* and *non-permanent* ones. For this study however, we will assume that all flow observations are permanent and aggregated per hour, which is consistent with the future expected situation.

- **Prior OD table.** This table is imported from the Dutch National Mobility Model. This matrix is updated periodically. Separate matrices exist for peak and off-peak periods and for weekdays and non-weekdays. However, the matrices are not available per hour.

- **Road networks.** A network containing the motorways and the major roads in the underlying network. No information on route choice proportions is available. Instead, routes are computed on the basis of link lengths assuming All-or-Nothing (AON) route choice behavior.

The following notation is used:

- \( T_{ij} \) cell in a priori matrix, containing daily traffic volumes from origin \( i \) to destination \( j \)
- \( y_{kt} \) average flow on link \( k \) in hour \( t \)
- \( q \) link for which a flow profile is to be estimated
\( \tau_{ijk} \) assignment map: \( \tau_{ijk}=1 \) if the route from \( i \) to \( j \) traverses link \( k \);
\( \tau_{ijk}=0 \) otherwise

The notation does not distinguish between types of day (Sun., Mon., etc.) or month. Day type and month are assumed constant in subsequent derivations.

3. **ANALYSIS**

A network can be thought of as a collection of \( N \) nodes and \( L \) links. The nodes are subdivided in \( N_1 \) origin nodes, \( N_2 \) destination nodes and \( N_3 \) internal nodes \((N_1+N_2+N_3=N)\). Due to the conservation of flow, this results in a maximum of \( N_3 \) linearly independent equations in the link flows. Therefore a minimum of \( L-N_3-W \) degrees of freedom exist, where \( W \) is the number of linearly independent flow observations. In addition to this it must hold that route flows are nonnegative. Generally, these conditions are insufficient to uniquely determine a set of link flows.

Therefore, additional assumptions are needed. Three options have been considered:
- Calibrating a distribution model on the observed volumes,
- Updating and assigning a prior matrix,
- Estimating local OD matrices.

### 3.1 Calibrating a Distribution Model on the Observed Volumes

A distribution model implies assumptions on the structure of the OD matrix that applies. Examples are models of the gravity or maximum entropy type. In theory it is possible to calibrate a distribution model from traffic counts. This is because the number of unknown parameters in a distribution model (e.g. the production and attraction abilities in a gravity model) is smaller than the number of links in a network.

Additional constraints may be added, e.g. one may assume that on a daily basis a symmetry in the flows exists as most trips are part of a chain starting and ending at the same point.

### 3.2 Estimating Local OD Matrices

Yet another option is to estimate local OD matrices from time series of traffic counts. This can be done using methods that are used to estimate dynamic OD tables (see e.g., Van der Zijpp, 1996). These methods extract turning proportions at local level from time series of traffic counts. In the future an option is to combine these traffic count data with data obtained from probe vehicles or surveys based on Automated Vehicle Identification (AVI).

### 3.3 Updating and Assigning a Prior Matrix

A more advanced approach is to update a prior OD matrix, using the available traffic counts. A distribution model may be used to obtain the prior matrix. However in the framework of the National Mobility Model such a prior matrix is estimated as well.
Using this matrix has the advantage that one benefits from the efforts that are spent building and checking this matrix.

3.4 Discussion and Choice of Method

After considering the options it was decided to investigate the option ‘Updating and Assigning a Prior Matrix’ (section 3.3) in more detail. This was done for the following reasons:

- The option based on calibrating a distribution model (section 3.1) has the advantage that no prior matrix is needed. However, the results are expected to be of poor quality especially if the ratio observed/unobserved links is low. Another problem is that the calibration is to be done for one-hour periods. Such a period length is short compared to the length of some longer trips. Moreover, a one hour period might not supply a sufficient aggregation level for a distribution model to apply.

- The option based on estimating local OD matrices from time series of traffic counts is conceptually attractive, but requires a lot of data to be applicable. The one-hour aggregated traffic counts were judged to be insufficient to apply such a method.

- The option based on updating a prior matrix utilizes existing information (the OD-matrix from the National Mobility Model) and seems easy to implement and to maintain. Various options are available for the updating mechanism. A method should be chosen that suits best with the objective to estimate link volumes rather than OD matrices.

4. ESTIMATION METHODS

A prior OD matrix can be updated and assigned at three levels of detail:

- Assigning prior OD matrix, no updating of OD cells. This is done by distributing the OD cell values over 24 hours and assigning the resulting matrices to the network. Flow observations are not used.

- Assigning partially updated OD matrix, only updating OD cells that contribute to observed link. Updating a prior matrix can be done in a number of ways. The most well-known techniques are the least squares technique and the Information Minimization technique (see Ortuzar and Willumsen, 1994). When applying least squares, only OD cells are updated that directly contribute to the observed link flows, other cells are left unchanged. For example, a prior matrix $T^0$ may be updated to $T$ as follows by using a set of link flow observations $K_{obs}$:

  \[
  T_{ij} = \arg\min_{f_{ij}} \left( \sum_{i,j} (T^0_{ij} - f_{ij})^2 + \sum_{k \in K_{obs}} (y_k - \sum_{i,j} f_{ij} \tau_{ijk})^2 \right)
  \]  

  (1)

  It can be shown that the cells of the resulting matrix $T$ only differ for those OD cells $T_{ij}$ for which $\tau_{ijk} \neq 0$ holds.
A similar argument applies to Information Minimization. The solution when applying IM satisfies:

\[ T_g = T_g^0 \prod_{k \in K_{\text{obs}}} X_k \tau_{gk} \]  

(2)

where \( X_k \) is a factor corresponding with the link volume measurements. Equation (2) shows that also in this case only cells contributing to the observed links, \( K_{\text{obs}} \), are updated.

- **Assigning fully updated OD matrix, updating all OD cells.** It may be argued that only updating cells contributing to observations, is not sufficient. It is likely that cells in OD matrices display a substantial positive correlation because the number of trips in different cells is influenced by many common factors, such as a common origin, destination or hour of departure. When adjustments need to be made to one group of cells in order to match a traffic count, it is likely that other cells need to be adjusted as well.

In order to implement a method based on a fully updated OD matrix, we express the unobserved link volumes in the observed ones. For this purpose we define a subnetwork in such a way that all OD paths traversing the subnetwork, cross at most one observed link *upstream* of link \( q \) and at most one observed link *downstream* of link \( q \). Note that ‘upstream’ and ‘downstream’ refer to routes. It is possible to construct a network and to select origins and destinations such that the upstream links of one route overlap with the downstream links of another route. However, for the networks considered for practical applications, such instances do not occur. Consequently, we may think of upstream links and downstream links as mutually exclusive sets.

Given a subnetwork, we may subdivide the OD flows that contribute to the flow on link \( q \) in the following categories:

- flows that contribute to observations *upstream*, but not *downstream* of \( q \),
- flows that contribute to observations *downstream*, but not *upstream* of \( q \),
- flows that contribute to observations both *upstream* and *downstream* of \( q \),
- flows that contribute to neither observations *upstream*, nor *downstream* of \( q \).

### 4.1 Estimation Framework

Given the categorisation of flows given above we may express the unknown flow profile as follows:

\[ y_{qt} = \sum_{r \in R_q} y_{rt} a_{rq} + \sum_{s \in S_q} y_{st} b_{sq} - \sum_{r \in R_q} \sum_{s \in S_q} y_{rt} c_{rq} + \nu_{qt} \]  

(3)

with:

- \( y_{rt} \) the flow on upstream link \( r \) during hour \( t \) \((t = 0,1,\ldots,23)\),
- \( y_{qt} \) the flow on link \( q \) during hour \( t \),
- \( y_{st} \) the flow on downstream link \( s \) during hour \( t \),
- \( R_q, S_q \) the sets of upstream and downstream observed links of \( q \),
\[ a_{rq} \] the proportion of the upstream observed volume \( y_{rt} \) that contributes to \( y_{qt} \),  
\[ b_{sq} \] the proportion of the downstream observed volume \( y_{st} \) that contributes to \( y_{qt} \),  
\[ c_{rsq} \] the proportion of the upstream observed volume \( y_{rt} \) that contributes to both \( y_{qt} \) and \( y_{st} \),  
\[ v_{qt} \] the number of vehicles that traverse link \( q \), but do not contribute to any observation; this number is referred to as the unobserved flow.

Equation (3) is still an exact expression. However, the local multipliers \( a_{rq} \), \( b_{sq} \) and \( c_{rsq} \) are not known, nor is the number of ‘unobserved’ vehicles, \( v_{qt} \). The following sections describe how values for these parameters can be estimated.

### 4.2 Computing the Local Multipliers

The multiplier \( a_{rq} \) indicates which proportion of the upstream measured volume \( y_r \) contributes to the unknown volume \( y_q \). This proportion equals the sum of OD flows contributing to both the flow on link \( r \) and the flow on link \( q \), divided by the total volume on link \( r \). This may be estimated on the basis of the prior matrix:

\[
\hat{a}_{rq} = \frac{\sum_{i,j} T_{ij} \tau_{ijr} \tau_{ijq}}{\sum_{i,j} T_{ij} \tau_{ijr}}, \quad r \in R_q
\]  

Likewise, estimates for the multipliers \( b_{sq} \) and \( c_{rsq} \) can be computed:

\[
\hat{b}_{sq} = \frac{\sum_{i,j} T_{ij} \tau_{ijs} \tau_{ijq}}{\sum_{i,j} T_{ij} \tau_{ijs}}, \quad s \in S_q
\]  

\[
\hat{c}_{rsq} = \frac{\sum_{i,j} T_{ij} \tau_{ijr} \tau_{ijs} \tau_{ijq}}{\sum_{i,j} T_{ij} \tau_{ijr}}, \quad r \in R_q \text{ and } s \in S_q.
\]

In this way the multipliers are computed without considering the type of day and the hour. In expression (3), these factors are multiplied with time-varying flow observations. As a consequence, a large part of the dynamics is still captured.

### 4.3 Estimating the Unobserved Flows

In absence of any other information, it is assumed that those flows contributing to the flow on link \( q \) but not to any upstream or downstream observation in \( R_q \) or \( S_q \), show the same variations with respect to their predicted values (according to the prior matrix) as exhibited by the observed flows.
The predicted value for the unobserved flows for a full day on the basis of the prior matrix can be computed as follows:

\[ \hat{v}_q = \sum_{i,j} \tau_{ijq}^R \tau_{ijq}^S T_{ij} \]  

(7)

with

\[ \tau_{ijq}^R = 1 - \sum_{r \in R_q} \tau_{ijr} \]  

(8)

and

\[ \tau_{ijq}^S = 1 - \sum_{s \in S_q} \tau_{ijr} \]  

(9)

where equations (8) and (9) denote the OD paths traversing link \( q \) while being unobserved upstream and downstream link \( q \), respectively.

The assumption of similar dynamic behaviour of both observed and unobserved flows implies that the estimated value for the unobserved flows \( v_{qt} \) in equation (3) is hence given by:

\[ \hat{v}_{qt} = \frac{y_{qt}^{RS}}{y_{qt}} \hat{v}_q \]  

(10)

with

\[ y_{qt}^{RS} = \sum_{r \in R_q} y_{rt} \hat{a}_{rq} + \sum_{s \in S_q} y_{st} \hat{b}_{sq} - \sum_{r \in R_q} \sum_{s \in S_q} y_{rs} \hat{c}_{rq} \]  

(11)

and

\[ y_{QT}^{RS} = \left( \sum_{i,j} T_{ij} \tau_{ijq} \right) - \hat{v}_q \]  

(12)

This completes the specification of the estimation method.

5. IMPLEMENTATION

In order to check the method, empirical data from the network surrounding Amsterdam are collected. Also an OD matrix obtained from the National Mobility Model (NMM) is used. This matrix is defined for 1349 zones. A number of practical difficulties exist:

- The size of the OD matrix is too big to allow for all manipulations that need to be done. Since the study area is restricted to the network surrounding Amsterdam, distant zones have been clustered proportional to their celestial latitude with respect to the study area: the closer these zones are to the study area, the less clustering has been performed. Zones within the study area were kept unchanged. By doing so, the number of zones has been reduced to 150. Consequently, a clustered OD matrix of \( 150 \times 150 \) cells has been obtained.

- The OD matrix is defined on the NMM network. This network differs slightly from the VLN network, containing the locations of the loop detectors. This problem has been tackled in four stages. First, the motorway part of the NMM network was cut out. Second, similar to the clustering of the OD zones, parts of
the VLN network far away from the study area, were simplified by removing crossing configurations and exit and entry links. Third, the simplified VLN network was embedded in the cut open NMM network. Finally, the left entry and exit links of the imbedded VLN network were connected with their nearest NMM zone centre nodes. Figure 1 shows the combined network. A detailed picture of the study area within this combined network is given by Figure 2.

- A matrix with the number of trips between origins and destinations is given, while a matrix with trips on entry-exit level of motorways is needed. Therefore, the OD paths traversing the study area, are determined - based on link lengths and assuming All-or-Nothing (AON) route choice behavior- and the contents of the corresponding OD cells are used to construct an even smaller prior OD matrix containing only amounts of trips entering, exiting or acting within the study area.

6. CONCLUSIONS AND FURTHER RESEARCH

In this paper we presented an approach to estimate flow profiles on links without detection loops, based on spatial dependencies between traffic flows traversing a subnetwork of upstream and downstream links. The link flow estimates consist of two parts: weighted sums of observed flows and estimated sums of unobserved flows. Note that the approach is semi static: local multipliers are determined by assigning a static OD table to the network; multiplying them with time-varying observations results in time-varying link flow estimates.

Although an All-or-Nothing route choice behaviour is assumed and hence the corresponding assignment map may be inaccurate, this matrix is only used to calculate relative local multipliers $\hat{a}_{rq}$, $\hat{b}_{rq}$ and $\hat{c}_{rq}$. Consequently, probable assignment errors will only slightly affect the resulting multipliers. The method is therefore expected to yield satisfying results, even in case of present congestion.

Time-varying unobserved flow estimations are obtained by assuming that variations with respect to the predicted values according to the prior matrix, are similar to variations apparent in the observed flows. To verify this assumption, the correlation between the corresponding cells of the prior OD matrix needs to be determined. If this correlation appears to be rather small, inaccurate estimates due to overcompensation may be found for links traversed by a relatively high amount of unobserved route flows. In that case, this part of the estimation method should be adjusted.

To assess the method’s performance, a proto-type applicable to the network around Amsterdam, is being developed. The implementation included adaptation of the NMM prior OD matrix by clustering zones outside the study area, simplification and combination of the NMM and VLN networks. For reasons of accuracy, it is recommended to repeat constructing such a combined network every five years so that the road changes and extensions to both networks during that period, are taken into account.
In order to obtain realistic reference data, the estimation method will initially only been applied to observed links. This means that one observed link at a time is regarded to be unobserved: its hourly flows are to be estimated while the available observations are used for comparison only.

Once the method has proved to be accurate enough, it will be used for choosing the optimal number and location of detection loops needed to estimate flow profiles for all motorway network links. Under the assumption of a prior OD matrix and some route choice behaviour (e.g., All-or-Nothing), this corresponds with a problem that can be uniquely solved. For practical purposes, a solution can be found by defining some upper bound to be satisfied for all links and denoting what ratio of the flows traversing these links, does not contribute to flows on any observed link.

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ABSTRACT

In the Netherlands a system exists for monitoring motorway usage. This is done by storing link profiles (represented by 24 hourly averaged volumes) for each weekday (Sunday, Monday, etc.). Link flow profiles are used for purposes such as planning road maintenance and identifying potential future bottlenecks. For part of the network, traffic is counted by means of detection loops. However, the majority of links is not equipped with these loops. Consequently, flow profiles for these unobserved links need to be estimated. This paper discusses various approaches for link flow estimation, using not only observed link flows, but also additional information such as a priori origin-destination (OD) matrix and assumptions on spatial dependencies of network flows.

The method discussed in detail, is based on fully updating and assigning a prior OD matrix. By constructing a subnetwork of upstream and downstream links, part of the unknown link flow can be expressed in terms of nearby observed link volumes. The remaining part, representing the number of vehicles that traverse the considered link but do not contribute to any observation, is estimated assuming a time-varying behaviour which is similar to that apparent in the observations.

To check the method’s applicability on a national scale, a prototype is being developed. The motorway network near Amsterdam is selected as study area. The required prior OD table is extracted from the National Mobility Model (NMM) and afterwards reduced in size by clustering zones lying far away from this study area. In addition, a suitable network is constructed by combining and simplifying the NMM network and the VLN network, containing the locations of the loop detectors.